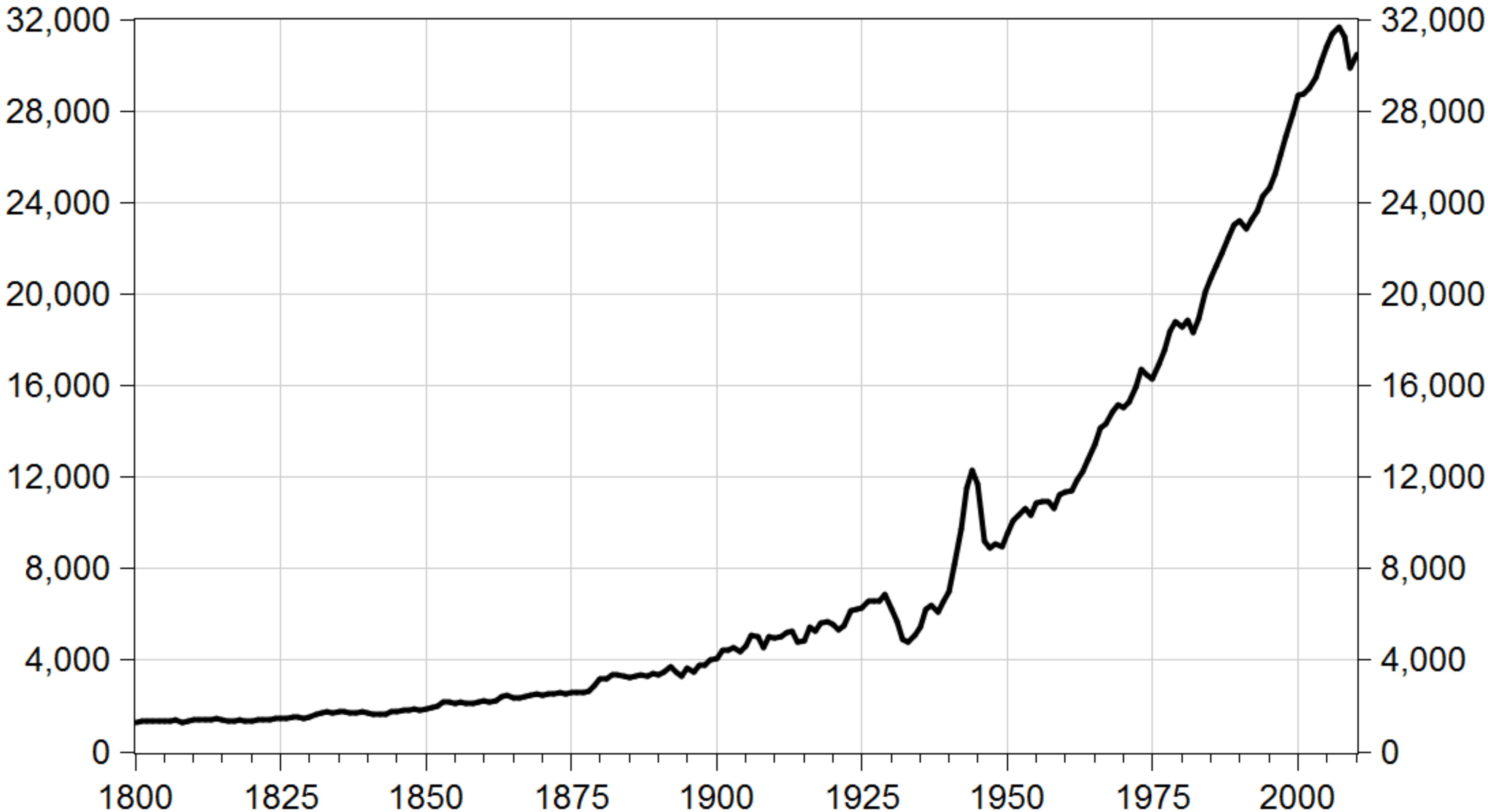
Time series

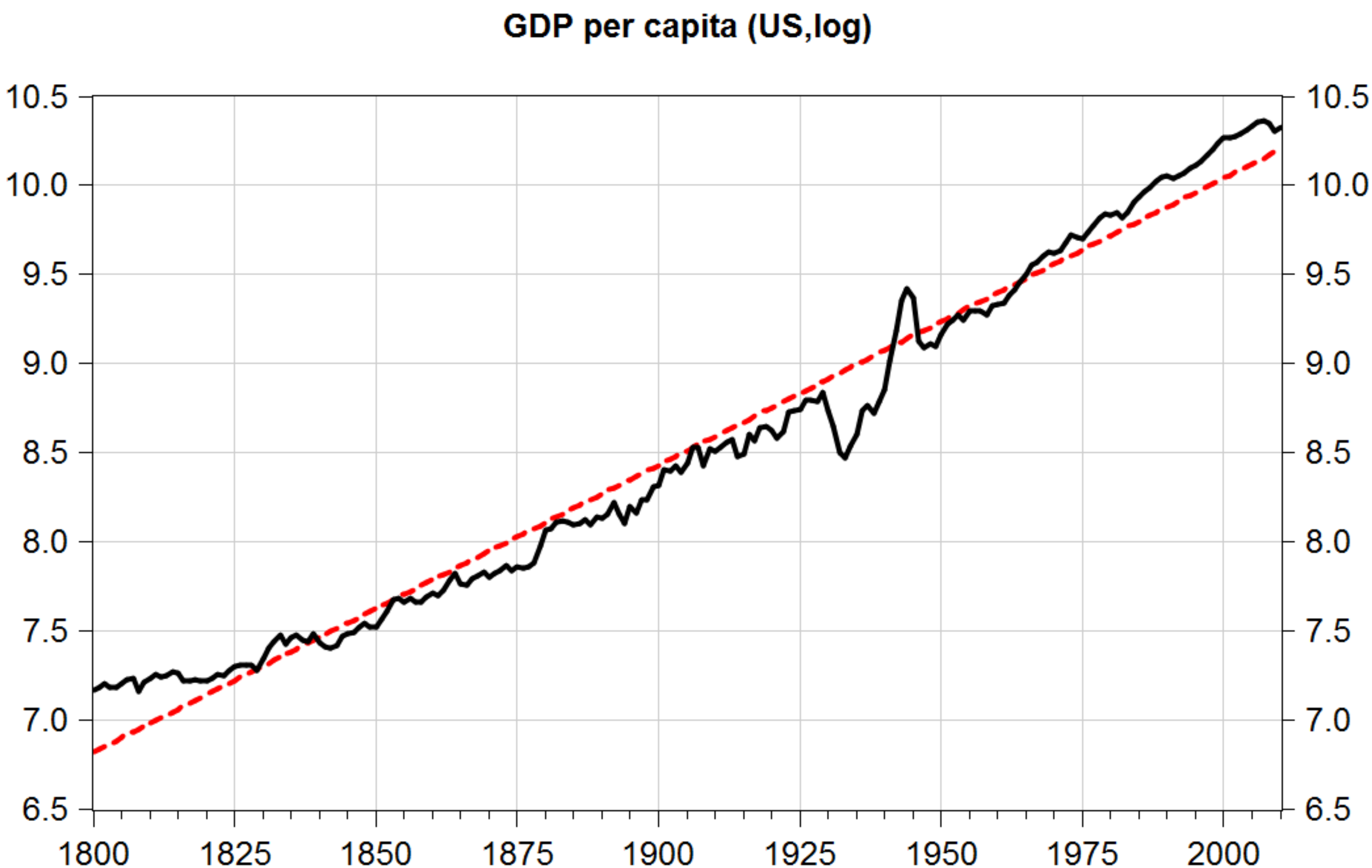
Lecture 4

Quantitative Tools level II

US GDP per capita



Source: Maddison



Source: Maddison

Cross-section vs. Time-series

- Cross-section / longitudinal : collection of observations for multiple subjects at single point in time. (eg: a drug trial).
- Time-series : collection of observation for a single subject at **regular intervals** over a long period of **time** (eg: GDP growth). **Time-dependent** signal.

Cross-section

- use samples in order to make inferences about the population
- often interested in variation in change processes

Time-series

- Decompose various cyclical components and trend processes
- Describing temporal dynamics
- Forecasting future time points

What are the main characteristics of time-series ?

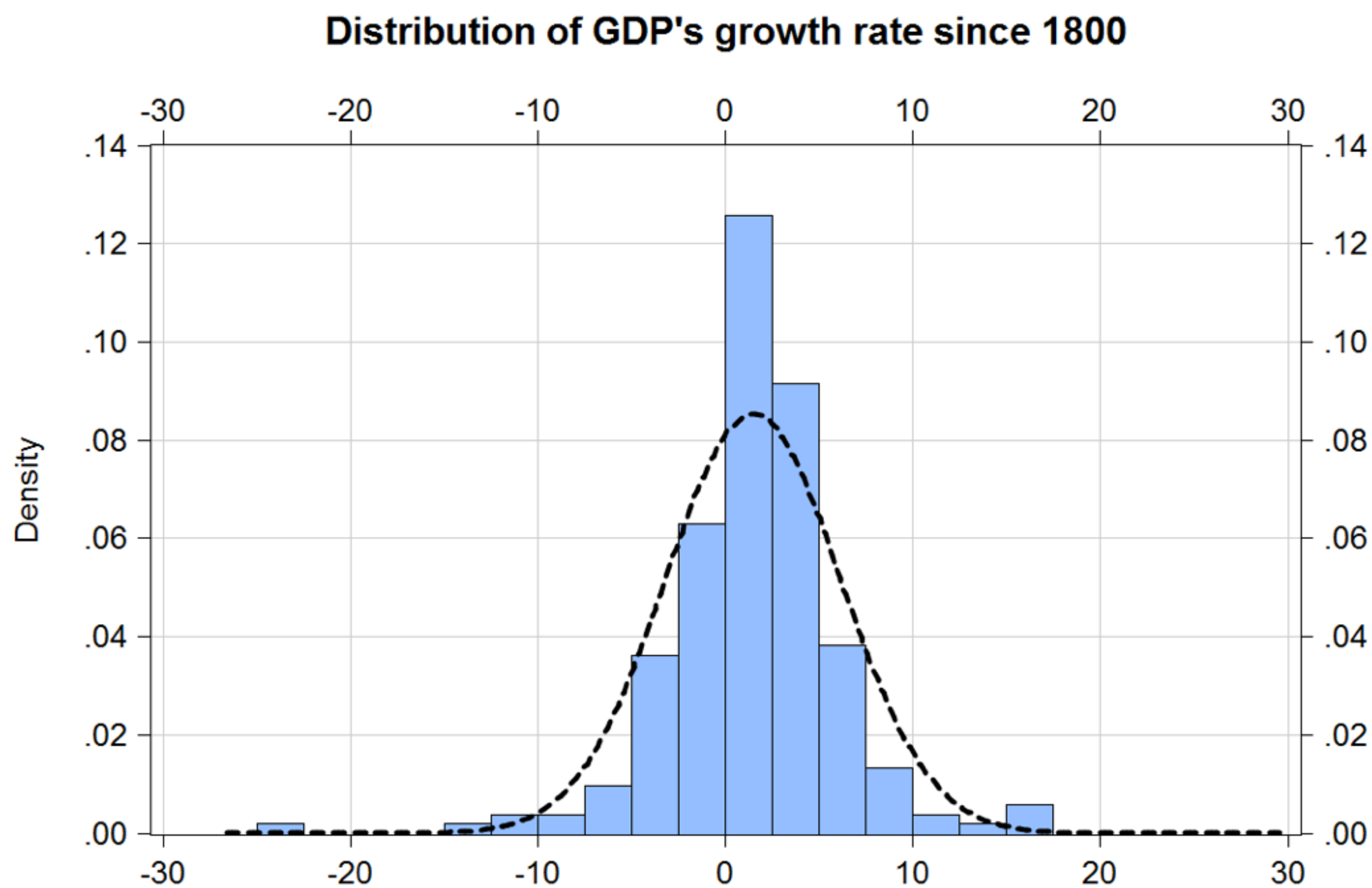
- Descriptive statistics : mean, median, st.dev
- Distribution
- Persistence, auto-correlations

Example - US GDP

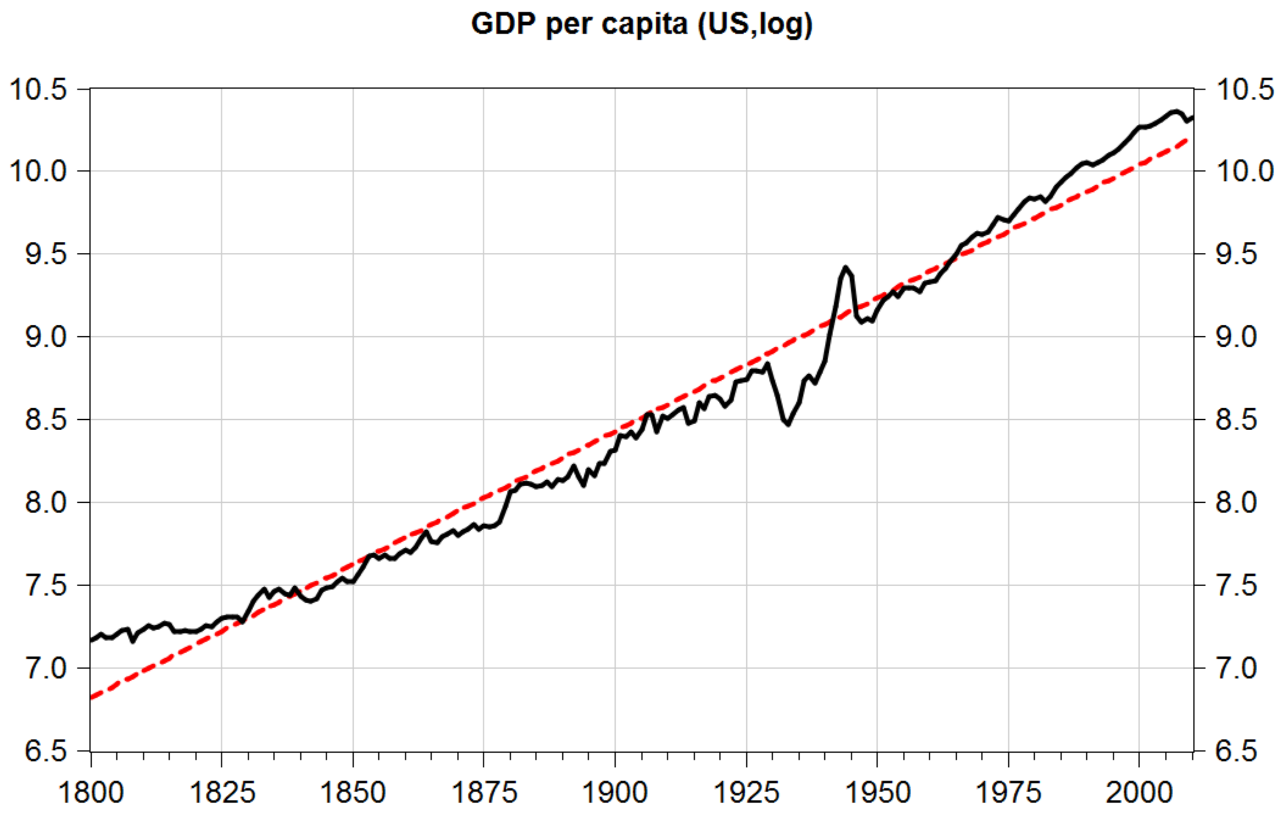
- Compute mean, median, standard-deviation.
- Draw the distribution of GDP *growth*.
- How is GDP growth at a given year related to GDP growth at the previous year.

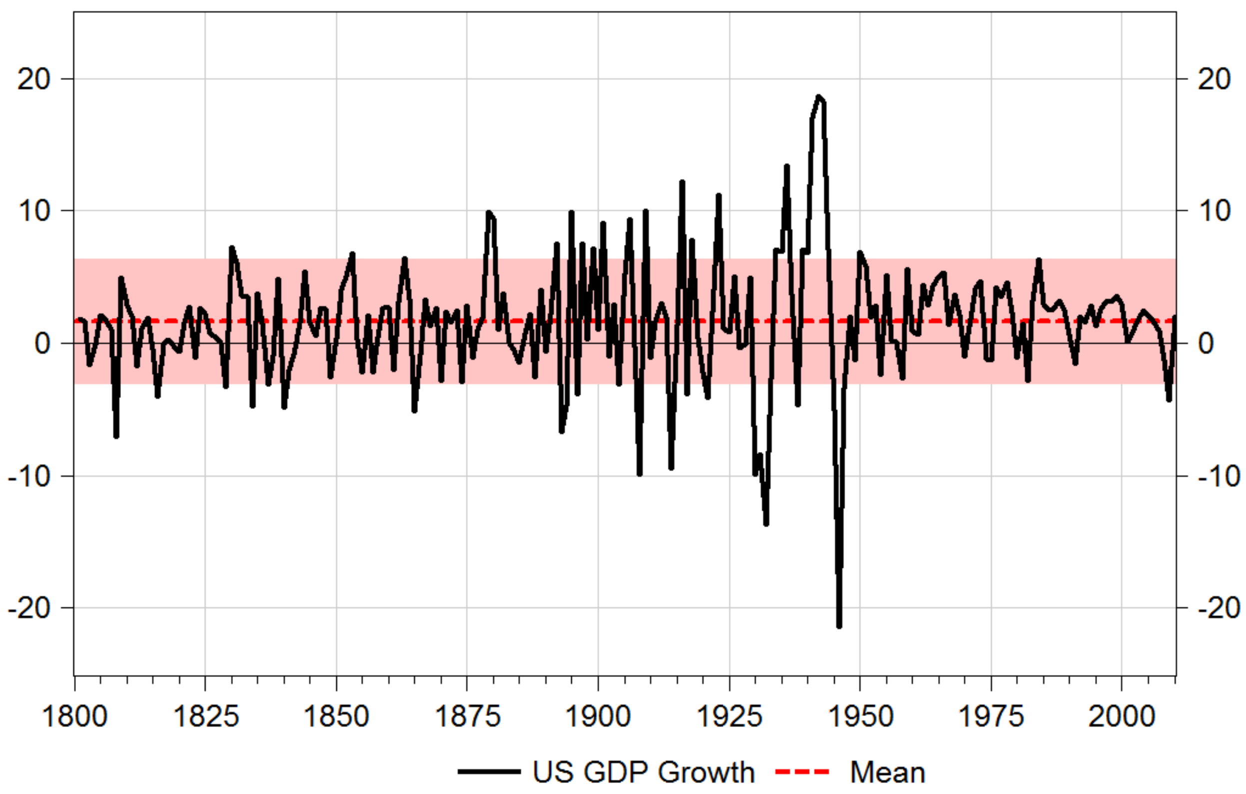
Example - US GDP

- Mean: 1.62% (/year)
- St.dev : 4.70pp



Example - US GDP





Example - US GDP

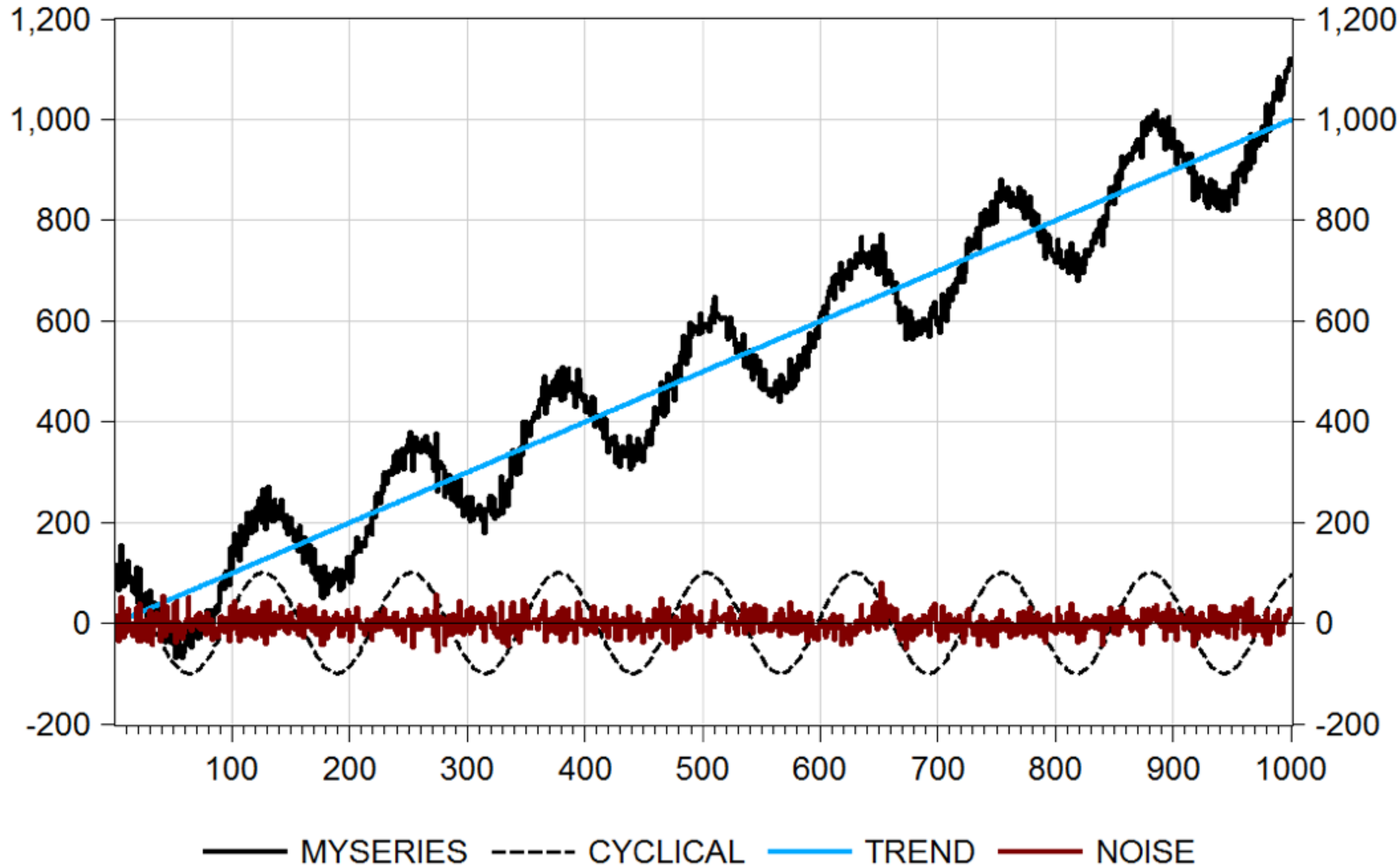
$$\Delta gdp_t = \alpha + \beta * \Delta gdp_{t-1} + \epsilon_t$$

Could you estimate β ?

A **time-series** usually has 4 components :

- a trend
- a cyclical component
- a seasonal component
- a random process (noise)

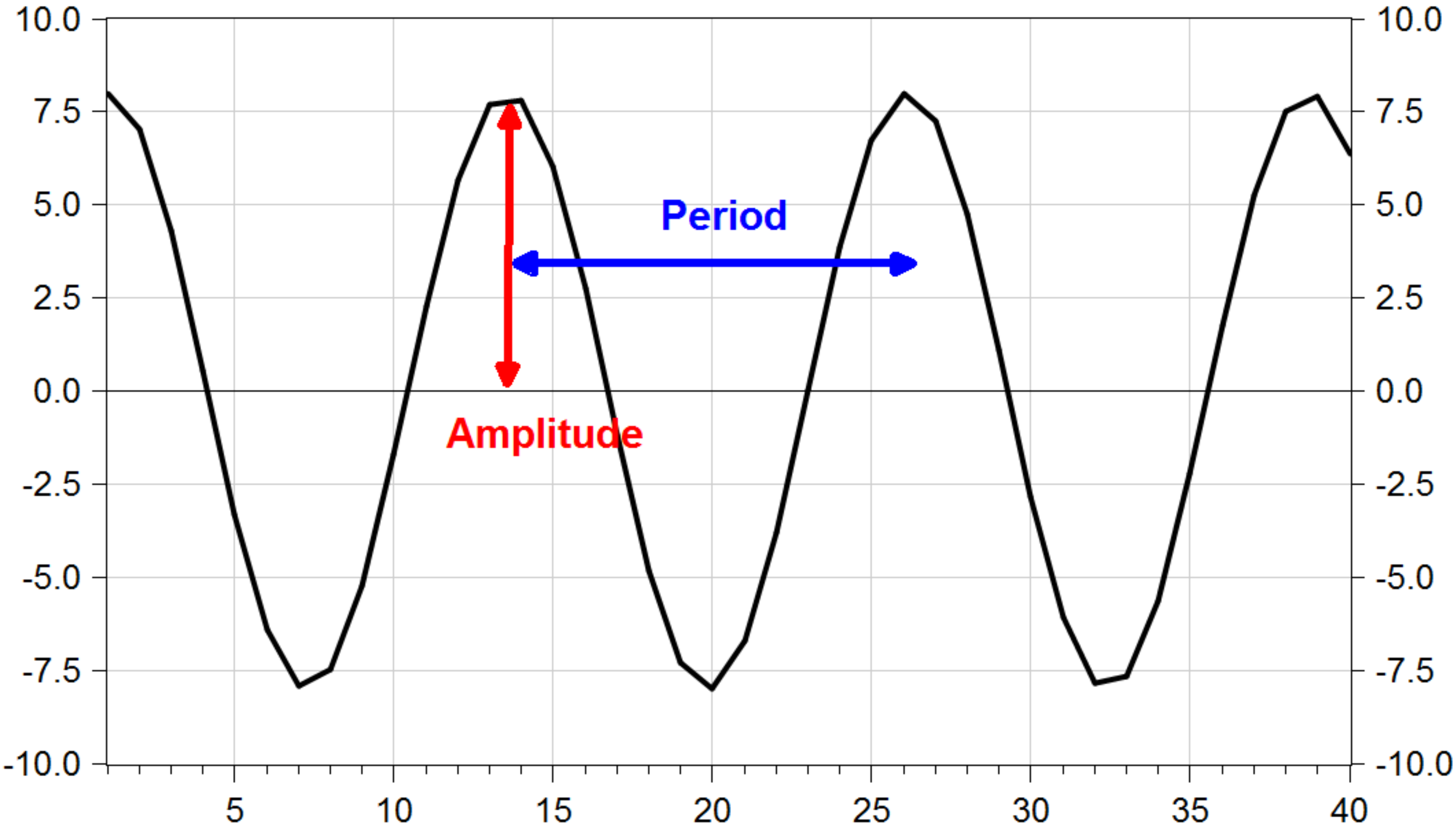
Split trend from cyclical component and noise



In Economics

- **Growth economics** : study the trend in growth
  - why has GDP been growing on average a 2% over two centuries ?
  - What is the role of technological change ?

- **Business cycles** : study the cyclical component :
  - Why are there recessions and expansions ?
  - What are the shocks hitting the economy ?



Period
Amplitude
10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
-10.0
5
10
15
20
25
30
35
40

How to split trend and cyclical components ?

- Simple methods : moving average, OLS estimation
- Complex methods : Hodrick-Prescott filter, Kalman filter

Two Approach

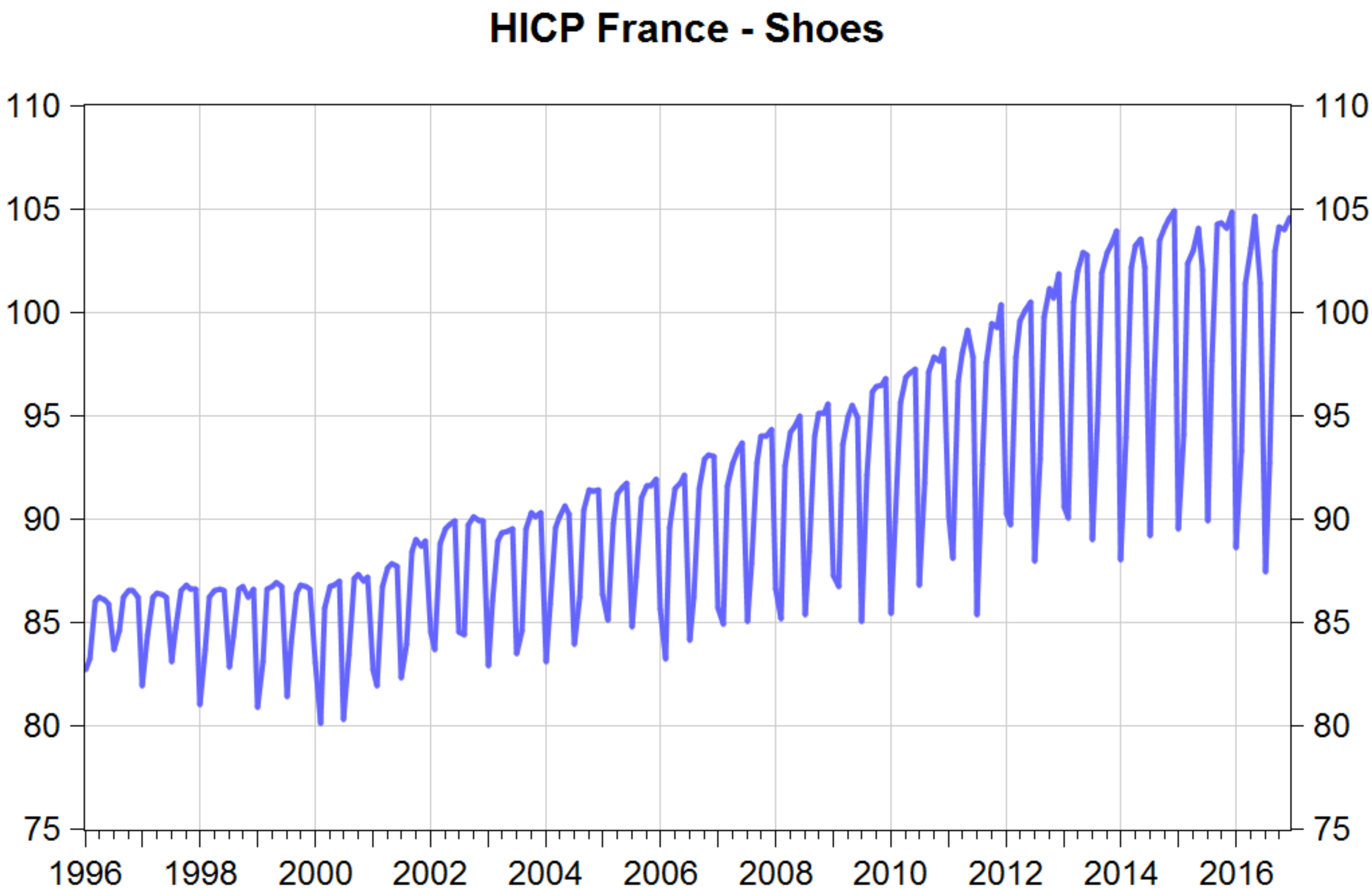
- Additive

$$y_t = t_t + c_t + s_t + \epsilon_t$$

- Multiplicative

$$y_t = t_t * c_t * s_t * \epsilon_t$$

How to seasonally-adjust data ?



Simple methods

- Ratio to mean (*only if time-series are not trended)*

$$q_m = \frac{1}{N}\sum_{n=1}^{N}\frac{y_{m,n}}{\frac{1}{12}\sum_{mo=1}^{12} y_{mo,n}}$$

- Ratio to trend (*only if time-series are not trended)*

$$q_m = \frac{1}{N}\sum_{n=1}^{N}\frac{y_{m,n}}{trend_{m,n}}$$

Simple methods

- Ratio to mean (*only if time-series are not trended)*

$$q_m = \frac{1}{N}\sum_{n=1}^{N} \frac{y_{m,n}}{\frac{1}{12}\sum_{mo=1}^{12} y_{mo,n}}$$

- Ratio to trend

$$q_m = \frac{1}{N}\sum_{n=1}^{N} \frac{y_{m,n}}{trend_{m,n}}$$

Simple methods

- Ratio to moving-average

$$q_m = \frac{1}{N} \sum_{n=1}^{N} \frac{y_{m,n}}{m.a.^{y}_{m,n}}$$

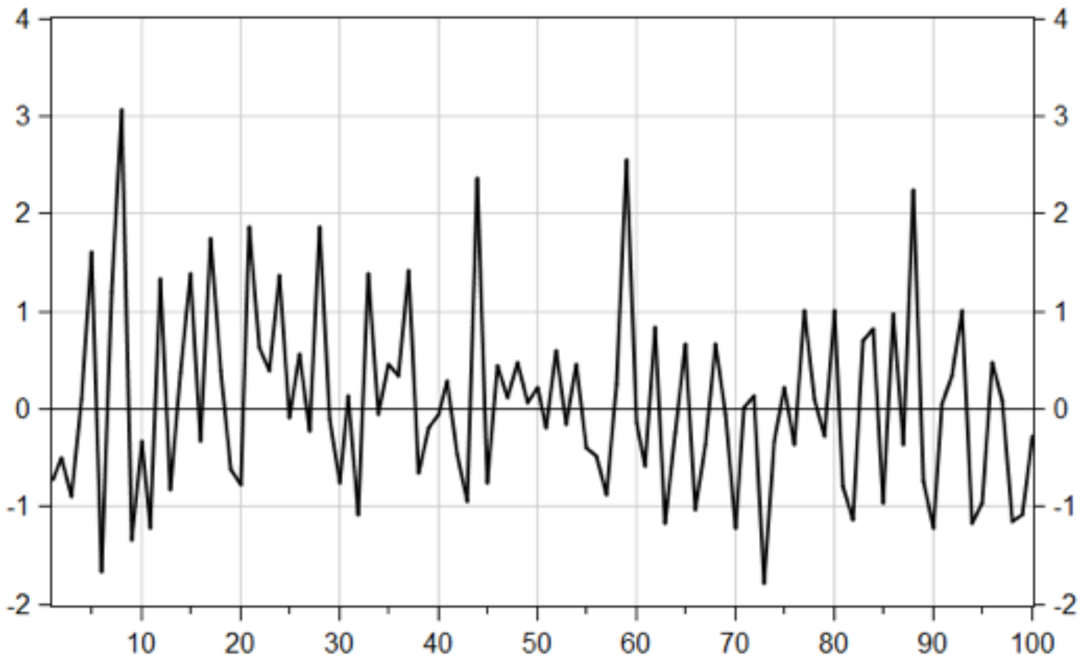
More sophisticated methods

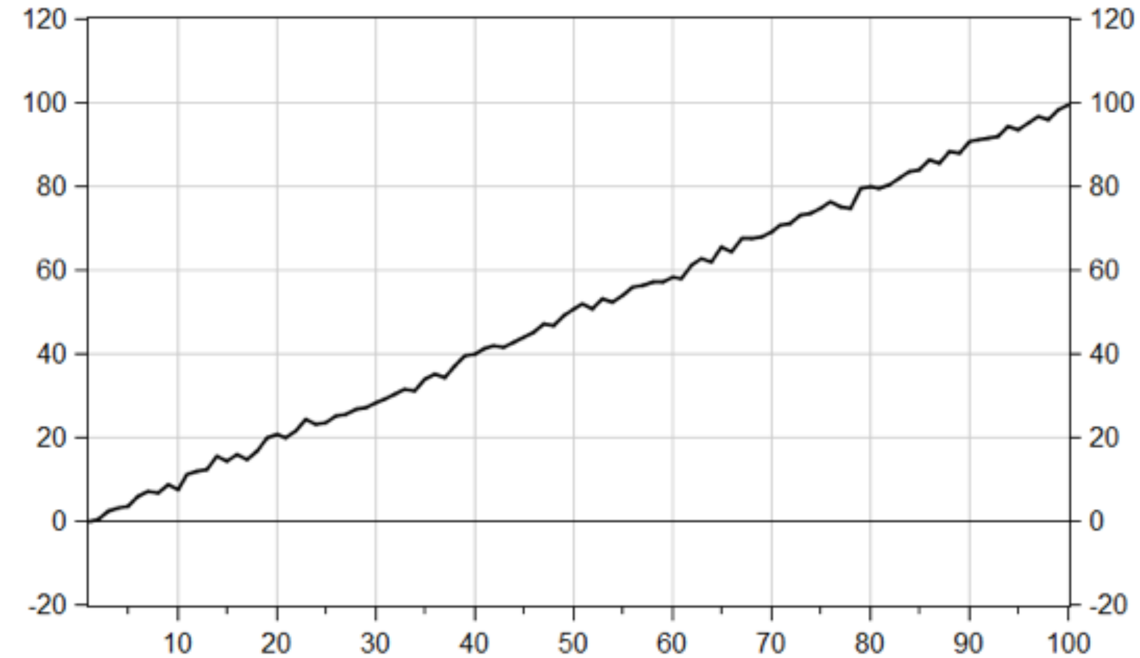
- ARIMA - X12
- Kalman filter

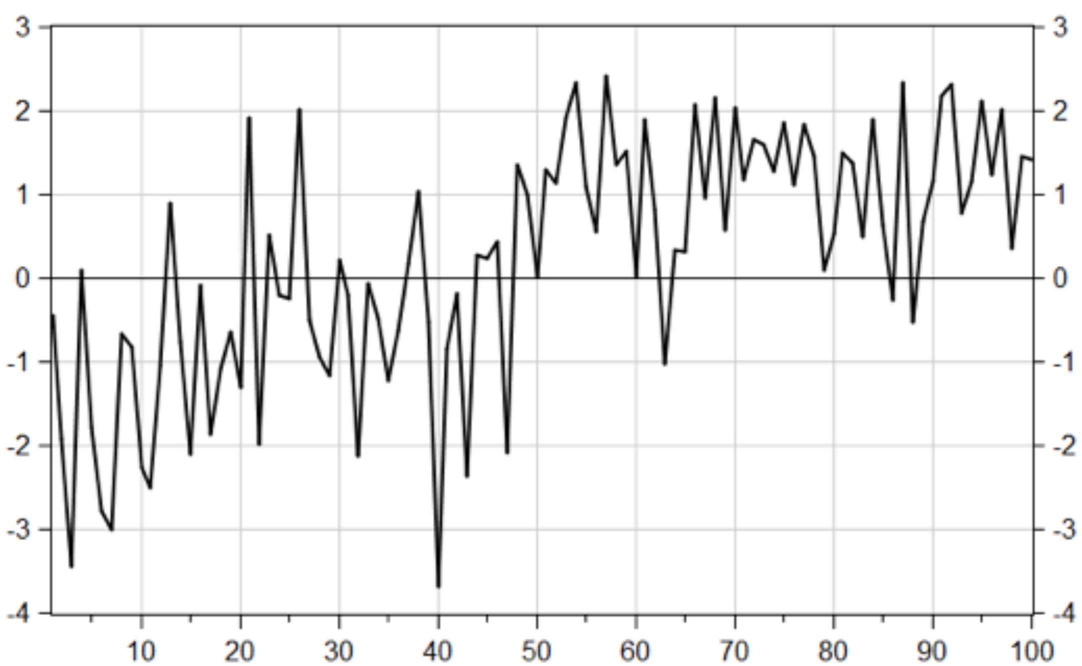
Stationarity

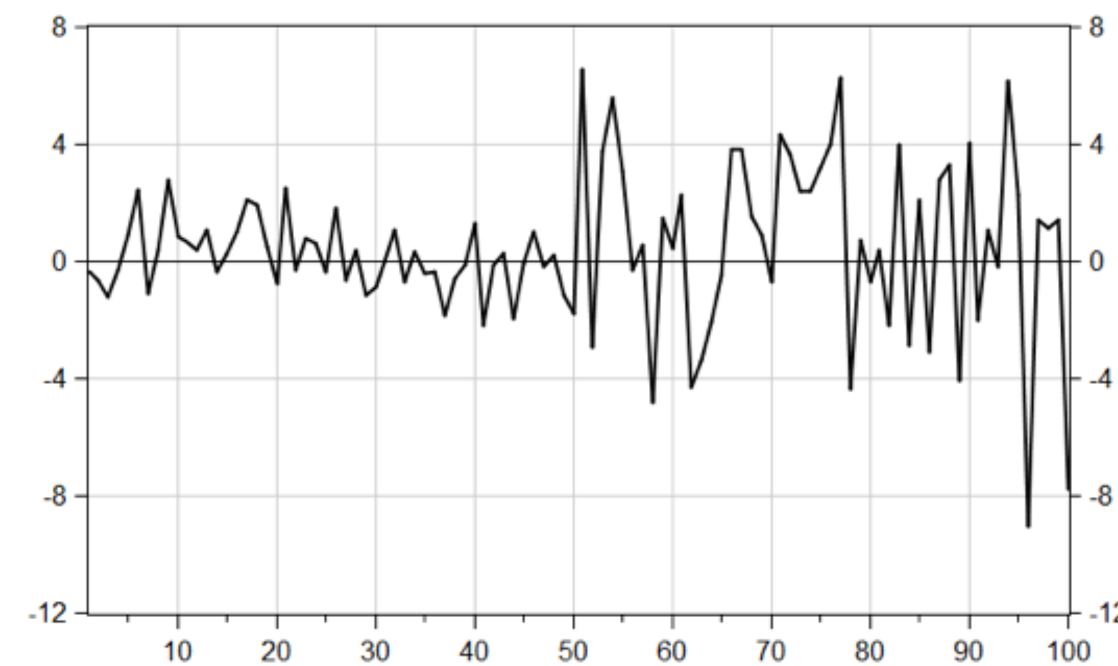
- Strong stationarity : distribution is invariant across time.
- Weak stationarity : mean and covariance are invariant over time

Which one is stationary ? (and why ?)





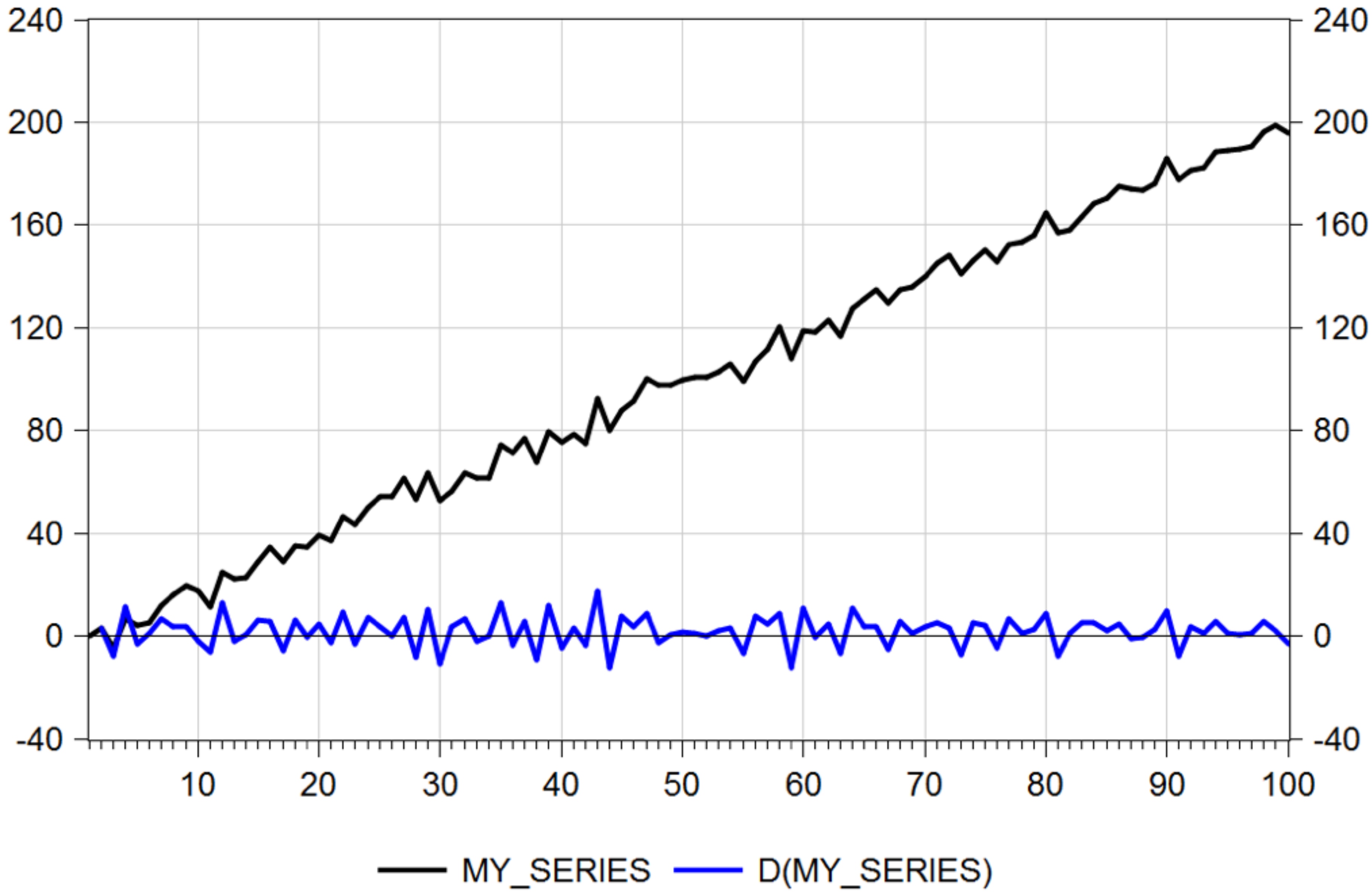




Examples of non-stationary processes
Trend data

$$Y_t = \alpha + \beta t + \epsilon_t$$

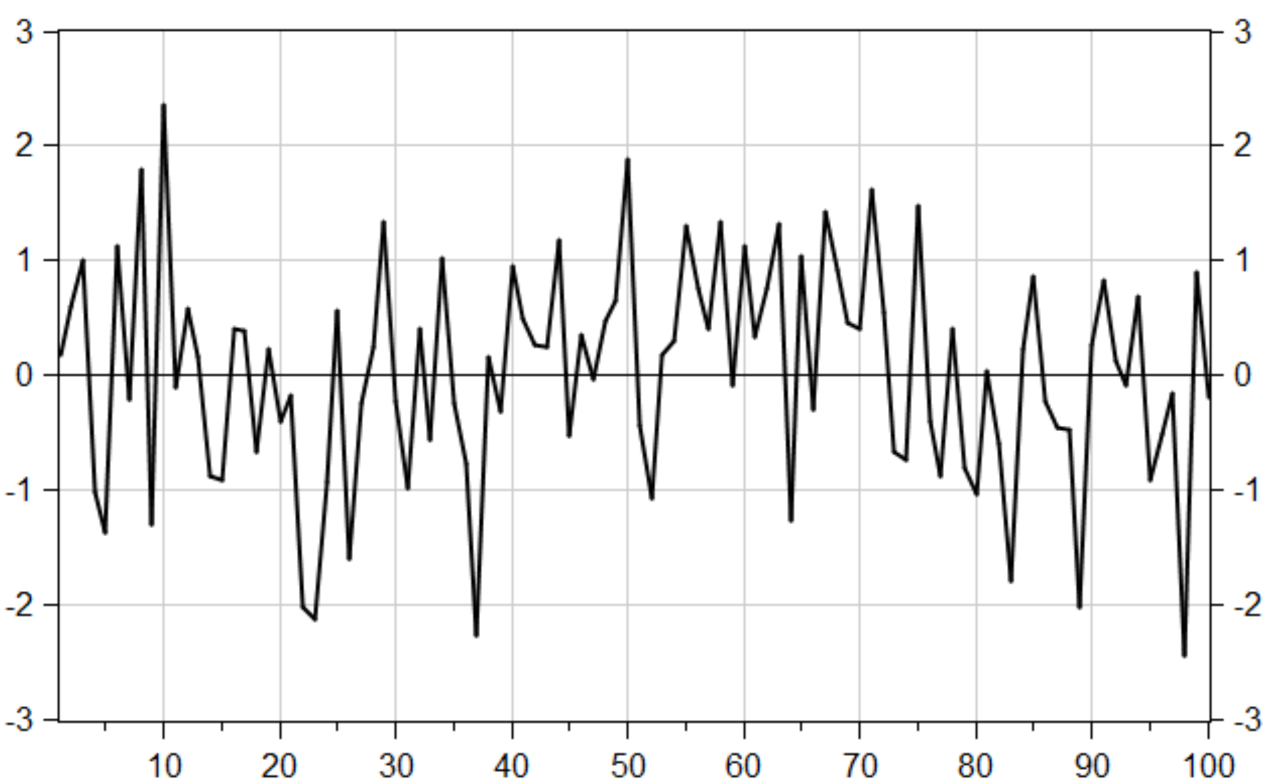
$$\Delta Y_t = \beta + \Delta\epsilon_t$$

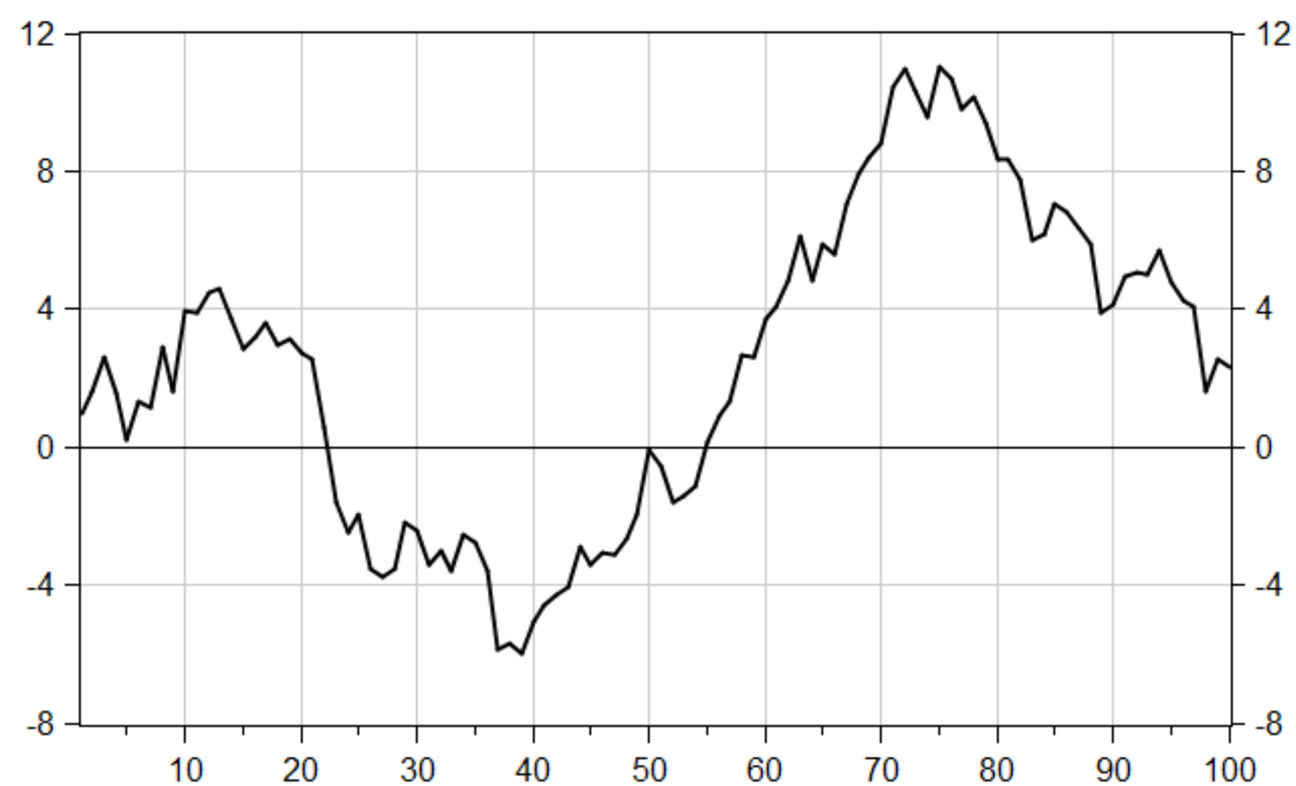


Examples of non-stationary processes
Random Walk

$$Y_t = Y_{t-1} + \epsilon_t$$

$$\Delta Y_t = \epsilon_t$$





Lag Operator

$$LX_t = X_{t-1}$$

$$L^2X_t = X_{t-2}$$

$$L^nX_t = X_{t-n}$$

Auto-Regressive Processes

A stochastic process which depends of its own lags

$$X_t = \mu + \sum_{j=1}^{p} \phi_j L^j X_t + \epsilon_t$$

MA Process

A stochastic process is a sum of past white noises

$$X_t = \mu + \sum_{j=1}^{p} \phi_j L^j \epsilon_t$$

Exercise : Estimate a Phillips Curve