Exercises - Probability (2/2)

Exo 1
In a class of 80 students, the professor calls on 1 student chosen at random for a recitation in each class period. There are 32 class periods in a term.

1. Write a formula for the exact probability that a given student is called upon j times during the term.

2. Write a formula for the Poisson approximation for this probability. Using your formula estimate the probability that a given student is called upon more than twice.

Exo 2
Reese Prosser never puts money in a 10-cent parking meter in Hanover. He assumes that there is a probability of .05 that he will be caught. The first offense costs nothing, the second costs 2 dollars, and subsequent offenses cost 5 dollars each.

1. Under his assumptions, how does the expected cost of parking 100 times without paying the meter compare with the cost of paying the meter each time?

Exo 3
A basket contains eight white balls and two black balls. We draw three balls from the basket without replacement. Let X denote the number of white balls drawn.

1. What is the law of X ?

2. What is $E(X)$ and $V(X)$?

Exo 4
A restaurant has three menus: A,B and C. Each customer chooses one and only menu among the three menus. We assume that each customer made a random, independent choice. Let A_n, B_n and C_n be the number of customers amid the n customers that showed up this day, that choose menu A, B, C respectively. For instance, A_3 denotes the number of people who have chosen menu A among the 3 customers that showed up this day.

1. Give the distribution of A_2 and compute its expected value and variance.

2. Give the distribution of A_n

3. Give the distribution of $n - A_n$

4. Infer the probability that all customers who show up on the same day will choose the same menu.

5. Find the probability that each menu is chosen at least once.

Exo 5
A royal family has children until it has a boy or until it has three children, whichever comes first. Assume that each child is a boy with probability 1/2.

1. Find the expected number of boys in this royal family and the expected number of girls.

Exo 6
A basket contains n red balls ($n \geq 2$) and five black balls. The game consists of randomly selecting two balls from the basket at the same time.

- If both balls are red, we win 10 euros.
- If one of the two balls is red, we win 2 euros.
- If neither ball is red, we lost 6 euros.

1. What is the probability, based on n, to win (i.e. earning money) ? Let G_n be the winnigs pocketed at the end of the game (depends of n).
2. What is distribution of G_n ?
3. Compute the expected value of G_n.
4. Find the value that n must take for the game to be fair (i.e. expected winnings are zero).

Exo 7
In a classroom, the height of students follows the normal distribution of mean $\mu = 170cm$ and standard deviation $\sigma = 10cm$. A student is chosen at random, find the probability that

1. his heights is greater than 190cm.
2. his height is lower than 160cm.
3. his height is within 160 cm and 180cm.

Exo 8
The Poisson distribution with parameter $\lambda = 0.2$ has been assigned for the outcome of an experiment. Let X be the outcome function. Find

1. $P(X = 0)$.
2. $P(X = 1)$.
3. $P(X > 1)$.

Exo 9
A typesetter makes, on the average, one mistake per 1000 words. Assume that he is setting a book with 100 words to a page. Let S_{100} be the number of mistakes that he makes on a single page.

1. What is the distribution of S_{100} ?

2. Find the Poisson approximation of S_{100} ?

Exo 10
Suppose that in a certain fixed amount A of blood, the average human has 40 white blood cells. Let X be the random variable which gives the number of white blood cells in a random sample of size A from a random individual.

1. What is the probability that X is lower than 38 or greater than 42 ?

Exo 11
Give the appropriate distribution for each of the following random variables:

1. Let X represent the roll of one die.
2. Let X represent the number of heads obtained in three tosses of a coin.
3. A roulette wheel has 38 possible outcomes: 0, 00, and 1 through 36. Let X represent the outcome when a roulette wheel is spun.
4. Let X represent the birthday of a randomly chosen person.