

**Paris School of International Affairs**  
**Outils quantitatifs niveau I**

**Examen**

**Mai 2016**

**Durée : 3 h**

*All calculators allowed; computers with an autonomous power supply are allowed. Most exercises can be done with a simple calculator and without a computer.*

*Students must do the exam in the language (French or English) of the class they attended.*

1. **(1 point)** A good is taxed at 20 % on the price excluding tax. What is the rate of the tax on the price including taxes?
  
2. **(2 points)** A proportional election “with a premium” follows the following electoral rule:  
The lead party, in percentage of votes expressed, has half the representatives and has, in addition, a percentage of the other half of representatives, in proportion to its percentage of votes cast.  
The other parties share the other half of representatives, in proportion to their percentage of votes cast.  
The votes cast are the number of registered voters, minus the abstentions and the blank or spoiled votes.  
There are three parties, A, B and C, and the number of representatives to elect is 400.  
Party A got the votes of 42% of registered voters.  
Party B got the votes of 24% of registered voters.  
Party C got the votes of 18% of registered voters.  
There are 16% of abstention and blank or spoiled votes.
  - a) What percentage of votes expressed did parties A, B and C obtain?
  - b) How many representatives does each party get? The result should be rounded to the nearest number.
  
3. **(2 points)** The table below presents the income tax rates.

Income bracket	0 - 100	100 - 400	400 - 1000	1000 - 2000	2000 - 5000	More than 5000
Tax rate	0%	10%	20%	30%	40%	50%

- a) How much will an individual who earns R=3000 pay in income tax?
  - b) The government decides that revenues between 100 and 400 will not be taxed anymore (tax rate: 0 %). What is the impact of this decision on the different taxpayers? Explain this impact qualitatively and precisely.
4. **(2 points)** The GDP of a country grew from 4000 monetary units to 4800 monetary units between year 1 and year 3. Between year 1 and year 3, i.e. over two years, the inflation rate was 10 %.
    - a) What was the overall real growth rate of GDP?
    - b) What was the average annual real growth rate of GDP?

5. (2 points) In a study on the salaries in a firm, the deciles ( $D_1, D_2, \dots, D_9$ ), the quartiles ( $Q_1, Q_2, Q_3$ ) and the median have been calculated. All the salaries are between 1000 and 10000.

Quantile	$D_1$	$Q_1$	$M_e$	$D_8$	$D_9$
Salary	1500	2500	4000	7000	9000

- a) Reconstruct the distribution of employees by « salaries bracket » and calculate the average salary.  
b) What is the standard deviation of this distribution?
6. (2 points) A bill with a nominal amount of 25 000€ is remitted for discount 49 days before its term. The discount rate is 4%.
- a) What is the commercial value (nominal value minus discount) of the bill?  
b) What is the rate of return (annual effective rate) of this discounting?
7. (2 points) A sum is invested with compound interests, and doubles in 10 years.
- a) How long will it take for the sum to triple (i.e. be multiplied by 3)?  
b) How long will it take for the sum to quadruple (i.e. be multiplied by 4)?
8. (5 points) Let's consider the following salary distribution in a company:

Salaries (€)	Number of people
from 10000 to less than 20000	200
from 20000 to less than 30000	800
from 30000 to less than 50000	180
from 50000 o less than 90000	20

Determine the following values and explain what they mean:

- a) The mean salary  
b) The median salary.  
c) The standard deviation and the coefficient of variation of salaries.  
d) The inter-decile ratio  $D_9/D_1$ .  
e) Draw approximately the Lorenz curve. Explain, using a graph (without any calculation), how one could determine the Gini Index.
9. (2 points) Let's study the effect of marketing expenses on the sales of a company. We will assume (to simplify) that marketing has an immediate impact on sales.

Year	1	2	3	4	5	6
Sales (K€)	18 000	19 200	19 800	20 400	20 000	21 600
marketing (K€)	104	108	116	124	128	140

- a) Determine the equation of the revenues as a function of the marketing expenses. You can use Excel if necessary.  
b) Give the value of the coefficient of determination  $R^2$  and interpret it.