Quantitative Tools - Level 2 - Fall 2017
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Lecture 3

Describing bivariate data (2/2)
Semi-log, Log-log, Elasticity

1 Linear and non-linear relationship

1.1 Intuition

So far we have studied *linear* relationship between variables, which is finding the best straight line that fits the data.
Although one could always estimate a linear relationship between variables, this is not always the best relationship, see the following example.

Example 1.1. Galileo's econometrics
In order to find how objects fall to Earth, a scientist throw a ball from the top of a tower and determine how the distance the ball falls is linked to the amount of time it falls. During the ball's fall, the scientist takes pictures to measure the distance covered at regular time intervals. The scientist *regress distance on time*, therefore estimating:

$$Distance = \alpha + \beta * Time + \epsilon$$

She finds a R^2 of 92.5, which is pretty high. However, plotting the data, she realised that it doesn't follow a straight line but a parabola (see Figure 2).

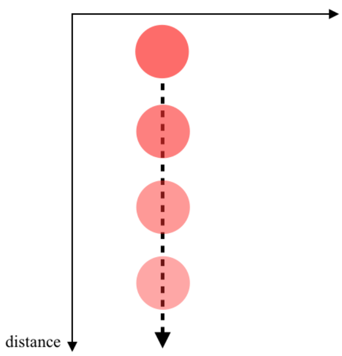

Figure 1: Trajectory of the ball

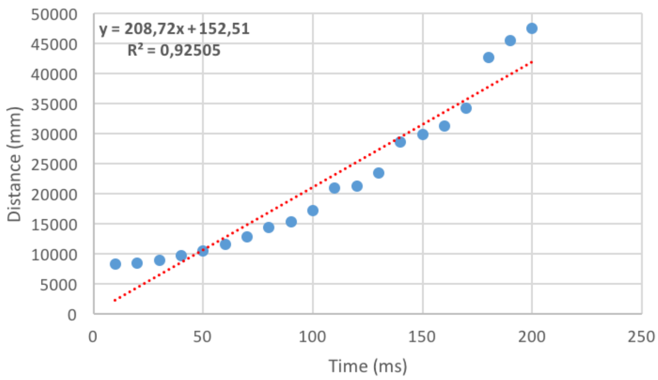

Figure 2: Plot

Moreover, the scientist has theoretical reasons to believe that distance is linked to the square of time. Based on theoretical reasons and her analysis of the data, she estimates the following equation:

$$Distance = \alpha + \beta * Time^2 + \epsilon$$

Note that the equation is still *linear* in the parameters, α and β (the equation has the functional form of a straight line) however it is not linear in the *variables*.

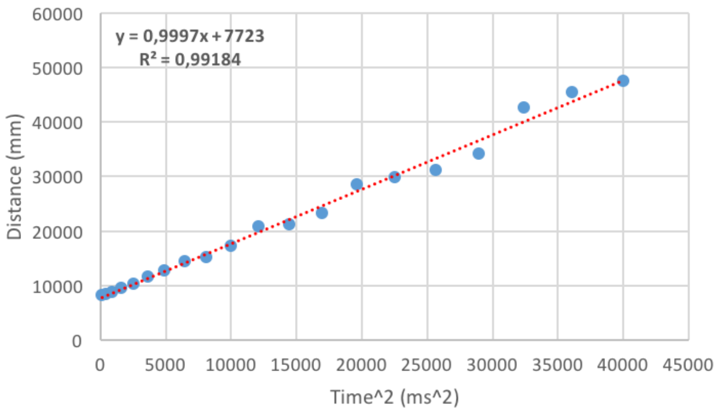

Figure 3: Plot

This model fits the data better that the previous one and indeed, the scientist finds a better R^2.

Linear regression is extremely flexible for describing data since it is possible to replace variables in the regression equation with functions of the original variables. For instance:

$$Y = \alpha + \beta X \tag{1}$$
$$Y = \alpha + \beta X^2 \tag{2}$$
$$Y = \alpha + \beta \ln X \tag{3}$$
$$\ln Y = \alpha + \beta \ln X \tag{4}$$

are all *linear regressions*, since they all remain in the form of an intercept plus a slope multiplying a (possibly transformed) X variable. In other words, linear regression means linear in the parameters, not in the variables.

Definition 1.1. Semi-log relationship
A *semi-log relationship* stands for the estimation of a equations which has the following functional form:

$$Y = \alpha + \beta \ln X$$

The variable Y is related to the logarithm of the variable X.

Definition 1.2. Log-log relationship
A *log-log relationship* stands for the estimation of a equations which has the following functional form:

$$\ln Y = \alpha + \beta \ln X$$

The logarithm of the variable Y is related to the logarithm of X.

1.2 Interpretation

How to interpret the **slope** of the linear regression ?

- in the case of **linear** relationship between variables, one can write:

  $$\Delta Y = \beta \Delta X$$

  which implies that a one unit change in X leads to β unit change in X.

- in the case of **semi-log** relationship between variables, we have:

  $$\begin{aligned} \Delta Y &= \beta \Delta \ln X \\ &= \beta \frac{\Delta X}{X} \end{aligned}$$

  which implies that a one *percent* change in X leads to β *unit* change in Y[1].

[1]When ΔX is small, $\ln \Delta X$ can be approximated by $\frac{\Delta X}{X}$.

- in the case of **log-log** relationship between variables, we have:

$$\Delta \ln Y = \beta \Delta \ln X$$
$$\frac{\Delta Y}{Y} = \beta \frac{\Delta X}{X}$$

which implies that a one *percent* change in X leads to β *percent* change in Y.

1.3 Elasticity

In the case of *log-log* relationship, the slope of the regression is

$$\beta = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}}$$

which is the elasticity of Y with respect to X.

Elasticity measures how a change in X (cause) affects Y (result). Usually, we use the concept for "price elasticity of demand" and we ask : by how much a change in the demand of a good responds to a change in the price of that good, computed as the percentage change in quantity demanded divided by the percentage change in price.

Applications

Elastic or inelastic good A good is said to be elastic if the quantity demanded responds substantially to a change in price. If it does not, the good is said to be inelastic. Thus, staple products like rice or bread are inelastic goods whereas Louboutin shoes and Picasso paintings are elastic goods. Usually, necessities have inelastic demand whereas luxuries have elastic demand.

Substitutes & Complements If the demand for good 1 increases with the price of good 2, then the goods are said to be substitutes. In the contrary, if the demand for good 1 decreases when the price of good 1 increases, then the goods are said to be complements. Thus, butter and margarine are substitutes whereas butter and honey are complements.
Formally, we talk about cross elasticity of demand and we compute :

$$\frac{\frac{\Delta Q_1}{Q_1}}{\frac{\Delta P_2}{P_2}} \begin{cases} > 0 & \text{goods are substitutes} \\ < 0 & \text{goods are complements} \end{cases}$$

Giffen good Usually, demand for a good decreases when its price increases (this is the law of demand). But there is a certain kind of goods for which this is not the case : basic necessities. When the demand of a good increases when its prices increases, we call it a **Giffen good**. The economic mechanism is as follows : if the price of rice rises in a poor countryside of China, people are likely to decrease their consumption of "luxuries" (like meat) to afford to buy more rice. And they will buy more to offset the meat deprivation. Thus, demand for rice is likely to increase.

There is another kind of goods for which demand increases with their price : luxury goods. They are called **Veblen goods**. The high price of that kind of good will be a sign of buyer's high social-status by way of conspicuous consumption. Thus, for Veblen goods, demand increases with their price.