Quantitative Tools - Level 2
Fall 2017

Exercises - Probability (1/2)

1 Sets

1. Let $A = \{1, 2, 4, 5, 6, 8\}$ and $B = \{0, 2, 4, 6\}$.
   - What is $A \cup B$?
   - What is $A \cap B$?
   - What is $\bar{A}$ in $A \cup B$?
   - What is $A \setminus A \cap B$?
2. Let A be the set of all possible outcomes of the experiment of rolling a dice. Let B the event "the dice falls on a odd number", and C the event "the dice falls on a number greater or equal to 3".
   - What is $card(B)$?
   - What is $card(B \cap C)$?
   - What is $card(B \cup C)$?
   - What is $card(B \setminus C)$?
3. (Hard) A finite set Ω has n elements. Show that if we count the empty set $\emptyset$ and Ω as subsets, there are 2^n subsets of Ω.

2 Combinatorics

1. Five people are to be arranged in a row to have their picture taken. In how many ways can this be done ?
2. A icecream shop has five different flavors. How many different two-balls icecream can he make ?
3. An automobile manufacturer has four colors available for automobile exteriors and three for interiors. How many different color combinations can he produce?
4. What is the probability that at least 2 of the French presidents have died on the same day of the year ? If you bet this has happened, would you win your bet [1]?
5. There are three different routes connecting city A to city B. How many ways can a round trip be made from A to B and back? How many ways if it is desired to take a different route on the way back?

[1] Hint: you should know how many presidents France ever had, there has been 25 presidents since 1848.

6. Five people get on an elevator that stops at five floors. Assuming that each has an equal probability of going to any one floor, find the probability that they all get off at different floors.

7. A certain state has license plates showing three numbers and three letters. How many different license plates are possible

   - if the numbers must come before the letters?
   - if there is no restriction on where the letters and numbers appear?

8. A family has 6 children.

   - What is the probability that there is one (and only one) girl ?
   - What is the probability that there is at least one girl ?
   - There are playing tennis two agains two. How many different teams can they make?

9. At the Euromillions lottery, a gambler should choose five numbers between 1 and 50 and two numbers between 1 and 9. How many chances do one has to win ?

3 Conditional Probability

1. (Monthy Hall problem) Suppose you're on Monty Hall's *Let's Make a Deal!* You are given the choice of three doors, behind one door is a car, the others, goats. You pick a door, say 1, Monty opens another door, say 3, which has a goat. Monty says to you "Do you want to pick door 2?" Is it to your advantage to switch your choice of doors?

2. A coin is tossed three times. What is the probability that exactly two heads occur, given that

   - the first outcome was a head?
   - the first outcome was a tail?
   - the first two outcomes were heads?
   - the first two outcomes were tails?
   - the first outcome was a head and the third outcome was a head?

3. A card is drawn at random from a deck of cards. What is the probability that:

   - it is a heart, given that it is red?
   - it is higher than a 10, given that it is a heart? (Interpret J, Q, K, A as 11, 12, 13, 14.)
   - it is a jack, given that it is red?