First Midterm - Spring 2016

Instructions

- The use of a mobile phone, or any other means of communication, is forbidden.

- Give all the results with a two decimals precision.

- Write answers on a separate sheet of paper.

Zamalek, Cairo 2011

1   Malek's Boutique (9 points)

Malek, a greengrocer, owns a small boutique in Zamalek, Cairo. The prices including taxes are the following :

Vegetables	Prices
Potatoes	2 EGP/kg
Eggplant	1 EGP/kg
Mango	4 EGP/kg

Malek is selling 10kg of potatoes a day, 10% more Eggplant and 20% less Mango than potatoes. The VAT is 15% and Malek makes a 20% markup (defines as the ratio of benefit and price without tax) on its sales.

1. For each item sold, Malek's profit is 20% of the price without tax. Thus the first thing to do is to compute the price without tax of each item. We know that the formula is

$$P_{with\,tax} = P_{without\,tax} * (1 + \tau_{VAT}) \tag{1}$$

So :

$$P_{without\,tax} = \frac{P_{with\,tax}}{1 + \tau_{VAT}} \tag{2}$$

We use that formula to get the price without tax for each of the item sold

Vegetables	Prices without taxes
Potatoes	1.74 EGP/kg
Eggplant	0.87 EGP/kg
Mango	3.48 EGP/kg

We know that Malek profit is 20% of the price without tax. So to get the profit made on each product, we multiply by 0.20 and we get

Vegetables	Profit
Potatoes	0.35 EGP/kg
Eggplant	0.17 EGP/kg
Mango	0.70 EGP/kg

So Malek profit on 1 sold kilogram of Mango is $0.70 EGP$

2. In order to compute Malek daily earnings, we multiply for each item the profit made per kilogram by the number of kilograms sold and we sum up. The quantities sold are :

Vegetables	Quantities sold
Potatoes	10 kg
Eggplant	11 kg
Mango	8kg

Thus the daily earnings are :

$$Profit = 0.35 * 10 + 0.17 * 11 + 0.7 * 8$$
$$Profit = 10.96 EGP$$

VAT increases from 15% to 16%. How does Malek earnings change if :

1. he does not change the price with tax

2. he does not change the price without tax

RESULTS

1. The profit is computed on the price without tax (it's 20% of the price without tax). So if the price without tax does not change, the profit is not going to change either. So the second option is straightforward : if Malek does not change the price without tax, his earnings will not change.

   If Malek does not change the price *with tax*, he should compensate the increasing taxes by reducing the price *without tax*. We use the first formula (see above) knowing that the price *with tax* does not change. To do it, we just divide each price with tax by $1 + \frac{16}{100}$. Once we do it, we multiply each price by 0.20 to get the new profit, multiply by the quantities sold and sum up. We found :

Vegetables	Profit
Potatoes	0.34 EGP/kg
Eggplant	0.17 EGP/kg
Mango	0.69 EGP/kg

And finally, the profit under the first condition is

$$Profit = 0.34 * 10 + 0.17 * 11 + 0.69 * 8$$
$$Profit = 10.86 EGP$$

So earnings of Malek would be reduced by 10c each day or by $\frac{10.86 - 10.96}{10.96} = 0.86\%$

Malek choose the second option and we now assume that the VAT rate is 16%.

5. Assuming that there are thirty days a month, what are Malek yearly earnings ?

6. The poverty line stands at 5,000 EGP in Egypt. By how much should Malek increase its daily earnings (in %) so that he reaches the poverty line ?

7. Malek increases its earnings by 10% each year. In how many years will he reach the poverty line ?

RESULTS

5. Malek has chosen the second option so its daily earnings are 10.96$. There are thirty days a month and twelve months a year, so there are $30 * 12 = 360$ days a year. Its yearly earnings are :

$$Yearly\,Profit = 360 * 10.96$$
$$= 3,944.35\,EGP$$

6. The poverty line is at 5,000 EGP, so Malek should increase its yearly earnings by :

$$Increase\,Rate = \frac{5,000 - 3,944.35}{3,944.35}$$
$$= 26.76\%$$

Malek should increase its yearly earnings by 26.76%, and its daily earnings by 26.76% also.

7. Malek will reach its poverty line when he will earn $5,000\,EGP$ each year. We know that each year its earnings increase by 10%, so each year its earnings are multiplied by 1.1. If we denote n the number of years so that Malek reaches the poverty line, we have :

$$3,944.35 * (1 + \frac{10}{100})^n = 5,000$$

We take the logs (as seen in class) and get :

$$n = \frac{ln(1.2676)}{ln(1.1)} \tag{3}$$
$$n = 2.49\,years \tag{4}$$

Inflation was 15% last year whereas Malek earnings only grown up by 10%.

8. How has Malek's purchasing power changed last year ?

9. Assuming inflation remains constant this year (at 15%), by how much should Malek increase its earnings so that average inflation over the two years equals the average growth rate of Malek's earnings ?

RESULTS

8. Prices have increased by 15% on average, so the multiplying factor of prices is 1.15, whereas the multiplying factor of Malek income is 1.1. Using the formula seen in class, Malek's purchasing power have changed by :

$$\frac{1.1}{1.15} - 1 = -4.35\% \tag{5}$$

9. In two years, prices will be multiplied by 1.15*1.15. We want to find x the increase rate of Malek's income so that $1.1*(1+\frac{x}{100}) = 1.15*1.15$. So :

$$1.1*(1+\frac{x}{100}) = 1.15*1.15$$
$$\frac{x}{100} = \frac{1.15*1.15}{1.1} - 1$$
$$x = 20.23$$

Thus, Malek should increase its earnings by 20.23% the second year in order to get the same average growth rate as inflation over the two years.

2   The last shall stay last (5 points)

The evolution of prices in Egypt between 2010 and 2013 had been the following :

	Potatoes	Eggplant	Mango	Lamb
2011	+5%	+10%	-1%	-2%
2012	+2%	+3%	2%	1%
2013	+1%	+4%	3%	3%

Consumption expenditure of households in Egypt split according to the following table

	Representative Household	Top Tier	Bottom Tier
Potatoes	30 %	20%	45%
Eggplant	30 %	20%	45%
Mango	30 %	20%	10%
Lamb	10 %	40%	0%

1. Compute the Paasche index (using 2010 as the baseline period) for the representative household

2. Compute the Laspeyres index (using 2010 as the baseline period) for the representative household, top tier households, bottom tier households

3. Compute inflation rate using the Laspeyres index for the three different types of household. What can you say on the evolution of purchasing power amid the different types of households ?

RESULTS

1. We do not have any quantities nor prices, so we cannot use the standard formulas for Laspeyres and Paasche indexes. But we got price indexes and weights, so we use the fact that :

   - the Paasche index is the weighted harmonic mean of indexes
   - the Laspeyres index is the weighted arithmetic mean of indexes

   So, first thing to do is to get the price index. We know that at the baseline period, index equals 1. We then compute the index of the following year using the evolution of prices.

	Potatoes	Eggplant	Mango	Lamb
2010	1	1	1	1
2011	1.05	1.10	0.99	0.98
2012	1.07	1.13	1.01	0.99
2013	1.08	1.18	1.04	1.02

   The harmonic mean is the reciprocals of the arithmetic mean of reciprocals. So here we just get the reciprocal (aka invert) of each index. We find :

	Potatoes	Eggplant	Mango	Lamb
2010	1	1	1	1
2011	0.95	0.91	1.01	1.02
2012	0.93	0.88	0.99	1.01
2013	0.92	0.85	0.96	0.98

2. We compute the weighted arithmetic mean for each year and take the invert

Year	Weighted average	Invert of the weighted average (Paasche index)
2011	0.96	1.04
2012	0.94	1.06
2013	0.92	1.09

3. The Laspeyres index is the weighted average of the index. We do it according to weights for each type of households and found[1]

	Representative Household	Top Tier	Bottom Tier
2011	1.04	1.02	1.07
2012	1.06	1.04	1.09
2013	1.09	1.07	1.12

   We conclude that inflation had been more severe for the Bottom Tier households, implying that inequality of purchasing power had surged during the past two years.

---

[1] Refer to the Excel file for calculations if you need to.

3   EGP black market (3 points)

One Egyptian Pound (EGP) worths 0.13 $US in financial markets (this is the nominal rate). Ikea sells a Billy bookcase at 645EGP in Egypt and at 69.99$ in US. We will use Billy Bookcase prices for a proxy of price levels in both US and Egypt.

1. What do you think of the EGP, is it overvalued or undervalued ?

2. How should the nominal exchange rate adjust (in %) so that the real exchange rate equals 1 ?

3. Malek is importing potatoes from Sudan and pays them in USD. If the EGP is devaluated by 10%, how would that impact Malek earnings ? (Hint : use the markup rate and the VAT rate to compute the costs of 1kg of potatoes in USD)

RESULTS

1. We use the price of Billy bookcase to determine the real exchange rate :

$$RER = \frac{69.99}{645 * 0.13}$$
$$= 0.83$$

The RER exchange rate is lower than 1 meaning that the Egyptian Pound is over evaluated on the financial markets. An other way to see it is simply to get the ratio of a the price of a billy bookcase in the US and in Egypt. We find 0.11, which is inferior to the nominal exchange rate, implying that *in reality*, one EGP worth less in terms of purchasing power : the nominal exchange rate would predict that you need $69.99/0.13 = 538.38\,EGP$ to buy a Billy bookcase in Egypt. Actually, you need more.

2. We want the real exchange rate to be 1, so using the formula above, we are looking for x so that :

$$RER = \frac{69.99}{645 * x}$$
$$= 1$$
$$x = \frac{69.99}{645}$$
$$x = 0.11$$

We should go from $1\$ = 0.13EGP$ to $1\$ = 0.11EGP$. So, the exchange rate should depreciate by :

$$\frac{0.11 - 0.13}{0.13} = -16.53\%$$

3. If the EGP is devaluated by 10%, that means that costs would increase by 10%. We know that costs represent 80% of the price without tax of potatoes. The price without tax of 1kg of potatoes is $1.74\,EGP$. The costs are thus $1.74 * 0.8 = 1.39EGP$. An increase by 10% means that costs are now $1.39 * 1.1 = 1.53$. The final price without tax is still 1.74. Given the new costs, the profit made on sold kilogram of potatoes is now $1.74 - 1.53 = 0.21$. We compute the final profit given the daily quantities sold by Malek. We find :

$$Profit = 0.35 * 10 + 0.17 * 11 + 0.7 * 8$$
$$= 9.57$$

The profit before that increase was 10.96, the change is thus $\frac{9.57-10.96}{10.96} = -12.70\%$

4   Malek empire (3 points)

A few years later, Malek has expanded his business. His income has surged to 15'000 EGP a year. The income tax is progressive according to the following table :

Income ranges		
0 - 5000	5000 - 10000	Above 10,000
0%	5%	15 %

Table 1: Income tax rates

1. What is the amount of taxes paid by Malek ?

2. For each new EGP earned by Malek, how much goes to Malek's pocket ?

   RESULTS

1. The amount of taxes paid by Malek is $5,000 * 0 + 5,000 * 0.05 + 5,000 * 0.15 = 1,000 EGP$

2. For each new EGP earned Malek will pay 15% on it, so $0.85 EGP$ goes into his pocket

   Malek has hired a dozen of employees and pays them according to the following table :

Wages	Frequencies
1000 - 2000	3
2000 - 3000	7
3000 - 4000	5
4000 - 5000	2

Table 2: Distribution of the wages in Malek's firm

3. What is the modal class ?

4. What is the median class ?

   RESULTS

1. the modal class is 2,000-3,000 (because it has got the highest frequency).

2. the median class is the class of the median person, which is the 9th one, belongings to the 2,000-3,000 class. The median class is thus 2,000-3,000.

5 Bonus questions - Malek leavened bread (+2 points)

Malek has decided to expand further his business and to open bakeries slack in his boutiques. In order to produce leavened bread, he is making a culture of lactic acid bacteria. Each day, the same proportion of bacteria is reproducing itself. When a bacterium is reproducing itself, it gives birth to a new bacterium which can also reproduce itself the next day. So if there is 4 bacteria and half of it reproduce itself, the next days there are $4 + 4/2 = 6$ bacteria. And the day after $6 + 6/2 = 9$ bacteria. We suppose that this proportion is constant in time and that half of a bacterium gives birth to an other half (so we have no problem with non-integers).

At the beginning of the experiment, Malek grows 100 bacteria in culture. At the end of the experiments, 30 days after, Malek lab has 175 bacteria.

1. Each day, what is the proportion of bacteria that reproduces itself ?

2. 15 days after the beginning of the experiment, how many bacteria does the lab have ?

RESULTS

1. We are looking for a proportion x so that $100 * (1 + x)^{30} = 175$. We find :

$$1 + x = \left(\frac{175}{100}\right)^{(1/30)}$$

$$x = \left(\frac{175}{100}\right)^{(1/30)} - 1$$

$$= 1.88\%$$

So each day, 1.88% of the bacteria reproduce itself.

2. Given that each day, 1.88% of the bacteria reproduce themselves, after 15 days we have :

$$Number\ of\ bacteria = 100 * (1 + \frac{1.88}{100})^{15}$$

$$= 132.29$$

***