Quantitative Tools - level I
Fall 2015
Lecture 3 - Indexes - RESULTS

1 Building a Price Index

Refer to the excel file for the results. If you have any questions, send me an email.

2 Bob and Alice Index

The table below represents the weights of each good in total consumption of Alice and Bob in year 1. The last column is the price variation between year 1 and year 2.
We know that Alice is earning $30,000 per year while Bob is earning $20,000 per year. Both have a 10% bonus at the end of the year.

Good	Bob	Alice	Relative variation
Hair-cut	25	60	+15%
Baguette	50	35	+5%
Cup of coffee	25	5	+2%

Table 1: Price & Quantities of goods consumed in Tataouine

1. We know that the Laspeyres index is the weighted mean of elementary indexes. The elementary indexes of prices in year 2, with year 1 as the baseline period (remember that an index in the baseline period equal 100) are :

Good	Elementary Index in year 1	Elementary index in year 2
Hair-cut	100	115
Baguette	100	105
Cup of coffee	100	102

Table 2: Elementary indexes of goods in Tatatouine

We then compute the mean of elementary indexes according to the weight of each good in Alice and Bob consumption. We find a Laspeyres index in year 2 with year 1 as a baseline period.
For Alice :

$$L_{y_1/y_0}^{Alice} = \frac{60 * 115 + 35 * 105 + 5 * 102}{60 + 35 + 5}$$
$$L_{y_1/y_0}^{Alice} = 110.85$$

For Bob :

$$L^{Bob}_{y_1/y_0} = \frac{25 * 115 + 50 * 105 + 25 * 102}{25 + 50 + 25}$$
$$L^{Bob}_{y_1/y_0} = 106.75$$

2. We construct a purchasing power index, written as PP for Alice and for Bob, equals to the ratio of their wage index and price index. In year 1, both the wage index and price index equal to 100. In year 2, the wage index for Alice and Bob equals to 110 (as they have a 10% wage increase). Thus :

$$PP^{Alice}_{y_1/y_0} = \frac{110}{110.85} = 99.23$$

$$\rightarrow \text{Purchasing power of Alice has changed by } \frac{99.23 - 100}{100} = -0.8\%$$

We do exactly the same for Bob :

$$PP^{Bob}_{y_1/y_0} = \frac{110}{106.75} = 103.04$$

$$\rightarrow \text{Purchasing power of Bob has changed by } \frac{103.04 - 100}{100} = 3.0\%$$

3 Trade between Orsenna and Farghestan

In year 1, a basket of goods costs \$10,000 in Orsenna (\$ in the Orsenna currency) and €30,000 in Farghestan (€ is the Farghestan currency). The nominal exchange rate is 1\$ for €3.
5 years later, prices have increased by 20% in Orsenna and by 50% in Farghestan. The nominal rate has not changed.

1. First, we compute the price of the basket of goods 5 years in later in Orsenna and Farghestan, using the multiplying factor of prices which equals to 1.2 for Orsenna and 1.5 for Farghestan.

	Price in Orsenna	Price in Farghestan
Year 1	\$10,000	€30,000
Year 6	\$12,000	€45,000

Table 3: Price of the basket of goods

The exchange rate is 1\$ for €3. So the basket of goods in Orsenna is $12,000 * 3 = e\,36,000$.

2. In Farghestan, the basket of goods costs $\frac{45,000}{3} = \$15,000$.

3. We compare the price expressed in the same currency of the basket of goods in Orsenna and Farghestan. The basket of goods is cheaper in Orsenna than in Farghestan, so people in Farghestan would prefer buying a good from Orsenna. In other words, Orsenna should export goods to Farghestan and trade go from Orsenna to Farghestan.

4. In year 1, consumers were indifferent to buy a good in Orsenna in Farghestan. For the purchasing power parity to remain constant, it still should be the case in year 6. So, we need \$12,000=€45,000, which is equivalent to 1\$ for € $\frac{12,000}{45,000}$ =€3.75.

   You could also use the formula seen in class which is :

$$x_a = y_b * \frac{(1+\pi_b)}{(1+\pi_a)}$$
$$= with \begin{cases} x_a & \text{the quantity of money in currency } a \text{ needed to buy } y_b \text{ of b currency} \\ \pi_a & \text{the inflation rate in country A with currency a} \\ \pi_b & \text{the inflation rate in country B with currency b} \end{cases}$$

   Here we have :

$$\begin{aligned} x_a &= 1 \\ y_b &= 3 \\ \pi_a &= 20\% \\ \pi_b &= 50\% \\ \rightarrow 1\$ &= 3 * \frac{1+0.5}{1+0.2} = 3.75 \text{ Euros} \end{aligned}$$

5. Orsenna accept to reevalute its currency by 10%. It means that with 1\$, you should be able to buy 10% more euros, that is, with 1\$, you should be able to buy $3 * 1.1 = 3.3$ euros.
   To know what Farghestan should do, let us reverse the perspective, with €1, we can now buy $\frac{1}{3.3} = 0.30\$$.
   To respect the purchasing power parity, we should be able to buy $\frac{1}{3.75} = 0.27\$$. Indeed, if it was the case, the basket of goods in Farghestan will cost $45,000 * 0.27 = 12,000\$$, which is the price of the basket of goods in Orsenna.
   Thus, the exchange rate should go from 0.3 to 0.27, which is a change by :

$$\begin{aligned} \frac{0.27 - 0.3}{0.3} &= -0.12 \\ &= -12\% \\ &\rightarrow \text{the Euro should be depreciated by } 12\% \end{aligned}$$

4 Elasticities

Years	Squid		Sweet pepper	
	Price	Quantities	Price	Quantities
Year 1	4	90	1	270
Year 2	7	60	2	300
Year 3	9	?	2	400

Table 4: Price & Quantities of goods consumed in Tataouine

1. We use the formula seen in class, which is the relative change of quantity demanded for squids divided by the relative change of price of squids, between Year 1 and Year 2.

$$E = \frac{\frac{60-90}{90}}{\frac{7-4}{4}}$$
$$= -0.44$$

The elasticity E of demand with respect to price is negative, it respects the law of demand : the demand decreases when the price increases.

2. In the formula used in the previous question we know everything except the quantity in year 3. Let Q_i be the quantity of squids consumed in Year i, and P_i the price of squids in Year i.

$$E = \frac{\frac{Q_3-Q_2}{Q_2}}{\frac{P_3-P_2}{P_2}}$$

We want to $Q_3 =$ something. So let us re-arrange the formula :

$$\frac{\frac{Q_3-Q_2}{Q_2}}{\frac{P_3-P_2}{P_2}} = E$$
$$\frac{Q_3-Q_2}{Q_2} = E * \frac{P_2}{P_3-P_2}$$
$$Q_3 - Q_2 = E * \frac{P_2}{P_3-P_2} * Q_2$$
$$Q_3 = Q_2 + E * \frac{P_2}{P_3-P_2} * Q_2$$

We now replace each variable by its value

$$Q_3 = 60 + \left(-0.44 * \frac{7}{9-7} * 60\right)$$
$$Q_3 = 52.4$$

3. We want to know what happens to the quantities of sweet peppers consumed when the price of squids increases while the price of sweet peppers remains constant (or vice-versa, what happens to the quantities of squid consumed when the price of sweet peppers increases while the price of squids remains constant). Between year 2 and year 3, the price of the squids increases while the price of sweet peppers remains constant. We note that quantity of squids consumed decreases which is simply the law of demand, and that quantity of sweet peppers increases. Thus, when the price of squids increases, *ceteris paribus*, the quantities of sweet peppers consumed increase, so squids and sweet peppers are **substitutes**.
   We can cross-check it by computing the cross-elasticity :

$$CrossElasticty = \frac{\frac{400-300}{300}}{\frac{9-7}{7}}$$
$$= 1.5$$
$$1.5 > 0 \rightarrow \text{Sweet peppers and squids are substitutes}$$