

**Paris School of International
Affairs**

Quantitative Tools level I

Examen

May 2015

Duration: 3 h

All calculators allowed; computers with an autonomous power supply are allowed. Most exercises can be done with a simple calculator and without a computer.

Students must do the exam in the language (French or English) of the class they attended.

In case of any difficulties understanding the French version of the exam, non-French speaking students are allowed to temporarily have a look at the English version.

1. **(1 point)** A good is taxed at 20 % on the price excluding tax. What is the rate of the tax on the price including taxes?

2. **(2 points)** A proportional election “with a premium” follows the following electoral rule:
The lead party, in percentage of votes expressed, has half the representatives and has, in addition, a percentage of the other half of representatives, in proportion to its percentage of votes cast.
The other parties share the other half of representatives, in proportion to their percentage of votes cast.
The votes cast are the number of registered voters, minus the abstentions and the blank or spoiled votes.
There are three parties, A, B and C, and the number of representatives to elect is 400.
Party A got the votes of 42% of registered voters.
Party B got the votes of 24% of registered voters.
Party C got the votes of 18% of registered voters.
There are 16% of abstention and blank or spoiled votes.
 - a) What percentage of votes expressed did parties A, B and C obtain?
 - b) How many representatives does each party get? The result should be rounded to the nearest number.

3. **(2 points)** The table below presents the income tax rates.

Income bracket	0 - 100	100 - 400	400 - 1000	1000 - 2000	2000 - 5000	More than 5000
Tax rate	0%	10%	20%	30%	40%	50%

- a) How much will an individual who earns $R=3000$ pay in income tax?
 - b) The government decides that revenues between 100 and 400 will not be taxed anymore (tax rate: 0 %). What is the impact of this decision on the different taxpayers? Explain this impact qualitatively and precisely.

4. **(2 points)** The GDP of a country grew from 4000 monetary units to 4800 monetary units between year 1 and year 3. Between year 1 and year 3, i.e. over two years, the inflation rate was 10 %.
 - a) What was the overall real growth rate of GDP?
 - b) What was the average annual real growth rate of GDP?

5. **(4 points)** We consider the firms in a given sector according to the number of employees.

Number of employees	Number of firms
0 – 70	2000
70 – 100	1000
100 – 500	800
500 – 1000	500
1000 – 5000	200

- a) What is the size of the modal firm?
 b) What is the size of the median firm?
 c) What is the size of the mean firm?
 d) We would like to study the concentration in this branch, by constructing a Lorenz curve. Describe, without doing the calculations, the table to build in order to construct this curve. Specify the x and y axis, and how to calculate them.
6. **(2 points)** In a study on the salaries in a firm, the deciles (D_1, D_2, \dots, D_9), the quartiles (Q_1, Q_2, Q_3) and the median have been calculated. All the salaries are between 1000 and 10000.

Quantile	D_1	Q_1	M_e	D_8	D_9
Salary	1500	2500	4000	7000	9000

- a) Reconstruct the distribution of employees by « salaries bracket » and calculate the average salary.
 b) What is the standard deviation of this distribution?
7. **(3 points)** The evolution of the turnover of a small firm, as well as the number of employees, over the last seven years is presented in the table below.

Year	Turnover (in tens of thousands of €)	Number of employees
1	15	8
2	20	14
3	21	16
4	25	22
5	28	24
6	32	30
7	41	40

- a) We consider that the number of employees is the known variable. What is the linear model giving the turnover as a function of the number of employees?
 b) Justify this linear model by calculating the coefficient of correlation and interpret the result.
 c) What turnover can be forecasted if the firm hired 20 additional employees?
8. **(2 points)** A bill with a nominal amount of 25 000€ is remitted for discount 49 days before its term. The discount rate is 4%.
- a) What is the commercial value (nominal value minus discount) of the bill?
 b) What is the rate of return (annual effective rate) of this discounting?
9. **(2 points)** A sum is invested with compound interests, and doubles in 10 years.
- a) How long will it take for the sum to triple (i.e. be multiplied by 3)?
 b) How long will it take for the sum to quadruple (i.e. be multiplied by 4)?